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Cosmic coincidences and relic neutrinos

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Abstract
A simple phenomenological description for the energy transfer between a variable cosmological constant (CC) and a gas of relic neutrinos in an expanding universe can account for a near coincidence between the neutrino and dark-energy densities to hold over a significant portion of the history of the universe. Although such a cosmological setup may promote neutrinos to mass-varying particles, both with slow and quick neutrino mass changing with the expansion of the universe naturally implemented in the model, it also works equally well for static neutrino masses. We also stress what sort of models for variable CC can potentially underpin the above scenario.

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Before 1997 there was a longstanding and hard-pressing problem in modern physics called the cosmological constant (CC) problem [1]. After a remarkable discovery of dark energy starting in 1997 nothing spectacular happened but the problem was dubbed the 'old' CC problem. Besides subsequent anthropic considerations, not much light has been shed since and the problem remains practically intact. At the same time, by the discovery of dark energy, at least two additional (and related) problems were also generated. In the first one, one should explain why the CC is small but nonzero (now called the 'new' CC problem). In the second one (the 'cosmic coincidence problem'), one should explain the near coincidence of the CC energy density (ρ_Λ) towards the predominant component in the matter energy density in the universe at present, i.e., cold dark matter (ρ_m) [2]. They are of the same order today, although vary in a completely different fashion over the history of the universe.

However, even the 'cosmic coincidence problem' itself seems to be more profound since one should also explain today's coincidences of ρ_Λ towards the rest of the components of the universe: ordinary matter (ρ_m), radiation (ρ_γ) and neutrinos (ρ_ν). Indeed, although they all redshift quite differently, they all become equal to ρ_Λ within redshifts of order of a few. Now, the bottom line is that, in distinction from other components, the past cosmological behaviours of ρ_Λ and ρ_ν are not much known about. Concerning dark energy, all we know is that it is redshifting at present with the equation of state (EOS) being very close to -1 [3]. The past cosmological behaviour of ρ_Λ could be to retain the same EOS in the past, thus creating a genuine 'cosmic coincidence problem' at present, or to track matter components by switching to its present EOS only in the recent past, thus creating the 'why now?' problem. However, this need not necessarily be so for neutrinos. Namely, past and present cosmological bounds on ρ_ν are quite weak, thus making a simple scaling $\rho_\Lambda \sim \rho_\nu$ (within a factor of 10^2–10^3) the most elegant resolution of the present coincidence between dark energy and neutrinos.

The first step towards the above explanation for the near coincidence between ρ_Λ and ρ_ν was undertaken in [4]. In the model [4], the dark sector included, besides the usual dark-energy sector given by a quintessence-like scalar ϕ, also a sector of relic neutrinos. The idea was to promote the sector of relic neutrinos to an almost undilutable quantity, as to be tightly bound with the ϕ-sector. For this purpose, since the neutrino number density n_ν in [4] scales canonically, the neutrino mass was promoted to a running quantity[1], thus scaling almost as n_ν^{-1}. The dark sector, although being composed of two sectors, acts as a single unified fluid.

Here, we present a far simpler model not including any extra dynamical degree of freedom (no quintessence-like scalars), in which dark energy and neutrinos constitute separate sectors, but with energy being transferred between them [6]. The main ingredient is variable but 'true' CC, with the EOS being precisely -1. The continuous transfer of energy between the CC and the gas of relic neutrinos (and vice versa, depending on the sign of the interaction term) can be conveniently modelled by the generalized equation of continuity

$$\dot{\rho}_\Lambda + \dot{\rho}_\nu + 3H\rho_\nu(1+\omega_\nu) = 0, \tag{1}$$

where overdots denote time derivatives, and the EOS ω_ν for nonrelativistic neutrinos in (1), can be safely disregarded for all practical purposes[2]. Now, let us make a specific *ansatz* for the energy transfer between the two components coupled through (1), which leaves the number density of neutrinos to dilute canonically, but promotes the mass of the neutrino to a running quantity, that is,

$$m_\nu(a) = m_{\nu_0} a^{\alpha}, \qquad n_\nu(a) = n_{\nu_0} a^{-3}, \tag{2}$$

where α is a constant, the subscript '0' denotes the present-day values, and the present value for the scale factor is set to 1. The solution for ρ_Λ can be represented in a simple form

$$\rho_\Lambda(a) = \frac{\alpha}{3-\alpha}\rho_\nu(a) + \rho_\Lambda^C, \tag{3}$$

where both ρ_ν and ρ_Λ now scale as $\sim a^{-3+\alpha}$ and ρ_Λ^C is the integration constant. Note that ρ_Λ^C represents the true ground state of the vacuum. Regarding equation (3), several comments are in order. If $\alpha > 0$, we are in the realm of decaying CC cosmologies ($\dot{\rho}_\Lambda < 0$), whereas $\alpha < 0$ means that the transfer of energy is from neutrinos to the CC ($\dot{\rho}_\Lambda > 0$). Since the cosmic matter budget today consists of not more than 2% of massive neutrinos [9], one concludes for the $\alpha > 0$ case that ρ_Λ^C should always be nonzero (and positive), unless α is fine-tuned to be very close to 3 (such large values of α are yet excluded, see below). On the other hand, for $\alpha < 0$, the first term in (5) is negative, and hence also the large and positive ρ_Λ^C is required.

Now we try to restrict the α-parameter by assuming the validity of the generalized second law (GSL) of gravitational thermodynamics. The GSL states that the entropy of the event horizon plus the entropy of all the stuff in the volume inside the horizon cannot decrease in time. In a sense, it is appropriate to do such an analysis here because we are dealing with

[1] The idea of mass-varying neutrinos was first considered in [5].

[2] Note that although at least two neutrino species are strongly nonrelativistic today, a phase-space distribution of relic neutrinos still retains its relativistic form even for masses much larger than those indicated by terrestial measurements, see, e.g. [7]. In addition, large neutrino mixing revealed in neutrino oscillation experiments may serve to conclude that chemical potentials for all neutrinos should be small [8]. Also, for the sake of simplicity, we restrict ourselves here to the case of one neutrino family as in [4]; the generalization to three families of neutrinos is straightforward.

cosmologies in which ever accelerating universes always possess future event horizons. The idea of associating entropy with the horizon area surrounding black holes is now extended to include all event horizons [10]. The easiest way to gain information on the α-parameter is by considering the change of entropy in the asymptotic regime, $a \gg 1$. In this case, one should also add the entropy of Hawking particles because it is conceivable that the CMB temperature will drop below the Hawking temperature after some time in the (distant) future [11]. Thus we have

$$\frac{\mathrm{d}S_{\mathrm{tot}}}{\mathrm{d}} \geqslant 0, \qquad S_{\mathrm{tot}} = S_{\mathrm{eh}} + S_{\mathrm{de}} + S_m + S_\nu + S_\gamma + S_{\mathrm{rgw}} + S_{\mathrm{Hawk}}, \tag{4}$$

where the particular entropies in (4) are of the event horizon, dark energy, matter, neutrinos, photons, relic gravitational waves and of Hawking particles, respectively[3]. What we find is that for $\alpha < 0$ it is impossible to satisfy the GSL, whereas for $\alpha > 0$ it is possible, provided that

$$\alpha \lesssim 3/4, \qquad m_\nu > \frac{\sqrt{\rho_\Lambda^C}}{M_{Pl}}. \tag{5}$$

Note that the second restriction is trivially satisfied for nonrelativistic neutrinos. Hence, the GSL prefers positive and less-than-one α's.

Further restriction on the α-parameter can be obtained by using the concept of effective EOS (for dark energy), as put forward by Linder and Jenkins [13]. The effective EOS for the variable CC whose interaction is phrased by (1) can be defined similarly as in [13]

$$\omega_{\mathrm{dark}}^{\mathrm{eff}} = -1 + \frac{1}{3}\frac{\mathrm{d}\ln\delta H^2(z)}{\mathrm{d}\ln(1+z)}, \tag{6}$$

where $1+z = a^{-1}$. Here, any modification of the standard Hubble parameter H is encapsulated in the term δH^2 (including ρ_Λ^C). For the model under consideration, one obtains

$$\omega_{\mathrm{dark}}^{\mathrm{eff}} = -1 + \frac{(1+z)^{3-\alpha} - (1+z)^3}{\left(\frac{3}{3-\alpha}\right)(1+z)^{3-\alpha} - (1+z)^3 + \rho_\Lambda^C\big/\rho_{\nu 0}}. \tag{7}$$

Even though α's as large as $\gtrsim 1$ are sustained by (7) because the ratio $\rho_\Lambda^C\big/\rho_{\nu 0}$ can be large, such large values for α would spoil the tracking behaviour at earlier times when the constant term in the denominator of (7) ceased to be dominant, and therefore the only acceptable values are $\alpha \ll 1$. By combining these arguments with the arguments from the GSL, one sees that slow mass variation over cosmological scales is preferable.

Note that with another *ansatz*, in which the total number of neutrinos in a comoving volume changes while retaining its proper mass constant,

$$m_\nu(a) = m_{\nu_0}, \qquad n_\nu(a) = n_{\nu_0} a^{-3+\beta}, \tag{8}$$

with β being a constant, the 'coincidence' $\rho_\Lambda \sim \rho_\nu$ is still maintained (simply make the replacement $\alpha \to \beta$ in (3)). The nice feature that ρ_Λ, as a solution of (1), always tracks ρ_ν, is maintained even when both the total number of neutrinos and their proper masses change, i.e.,

$$m_\nu(a) = m_{\nu_0} a^\alpha, \qquad n_\nu(a) = n_{\nu_0} a^{-3+\beta}. \tag{9}$$

In this case, one can easily check that the effective EOS is still given by (7), but now with the replacement $\alpha \to \alpha + \beta$, and therefore the requirement $\alpha \ll 1$ turns into $\alpha + \beta \approx 0$, signalling

[3] Note that although we are dealing with the 'true' CC having $w_\Lambda = -1$, its entropy inside the event horizon S_{de} does not vanish because our Λ is a varying quantity. Furthermore, S_{rgw} is about to vanish at some time in the distant future, see, e.g. [12]. For details of such a type of calculation as well as proper definitions for particular entropies, see [18].

thus a two-way energy transfer between Λ and relic neutrinos. In this way we obtain quick neutrino mass changing with the expansion of the universe.

We would like to mention that the variable CC model [14] is completely able to underpin the present scenario. It is a decaying CC model with $\alpha + \beta > 0$. The model is based on the renormalization-group (RG) evolution for ρ_Λ, and on the choice for the RG scale $\mu = H$. It can be shown that a canonical value for $\alpha + \beta$ is $(4\pi)^{-1} \ll 1$. In addition, the CC-variation law, $\mathrm{d}\rho_\Lambda/\mathrm{d}z \propto \mathrm{d}H^2/\mathrm{d}z$, is a derivative one, thus having a natural appearance of a nonzero ρ_Λ^C. Finally, we consider a dynamical CC scenario generically dubbed 'holographic dark energy' (HDE). Derived originally for zero-point energies, the Cohen *et al* bound [15] for ρ_Λ can be rewritten in the form

$$\rho_\Lambda(\mu) = \kappa \mu^2 G_N^{-1}, \tag{10}$$

where μ denotes the IR cut-off and κ represents a degree of saturation of the bound. This is a very important concept since for natural values for κ of order of unity, the HDE model also represents one of the most elegant solutions of the 'old' CC problem. Through the relationship between the UV ($\rho_\Lambda \sim \Lambda^4$) and the IR cut-off as given by (10), the holographic information is consistently encoded in the ordinary quantum field theory. The most natural and simplest possibility is to have $\mu = H$. In our case, however, $\mu \sim H$ unavoidably implies $\rho_\Lambda^C = 0$, thus spoiling the successfulness of the scenario[4]. Still, agreement is possible for noncanonical choices for μ or modification of the law (10), for instance, by promoting κ [16] or G_N [17] to a varying quantity.

Let us summarize our main results and conclude by a few additional comments. The variable part of ρ_Λ dilutes at the same rate as ν's and hence we have $\rho_\Lambda \sim \rho_\nu$ for a large portion of the history of the universe. One the other hand, the tracking behaviour without the constant part in ρ_Λ leads to an unrealistic model of the universe. For $\alpha \ll 1$, we also have an approximate tracking of ρ_Λ, ρ_ν and $\rho_m \sim a^{-3}$, because in our scenario matter dilutes canonically. Note that the positivity of α is essential here to have dominance of matter over neutrinos in the past. If the interaction between Λ and ν's is maintained in the epoch where ν's become relativistic, one can be easily convinced, considering a replacement $-3+\alpha \to -4+\alpha$ in the above expressions, that then we have an approximate tracking of ρ_Λ, ρ_ν and $\rho_\gamma \sim a^{-4}$. The scenario is generally underpinned by renormalization-group running cosmologies. In its simplest and originally derived form, the scenario is not generally underpinned by HDE, but fits the generalized HDE models.

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